Topological Properties of the Lowest Landau Level: A Study of Chern Number Statistics

Edward Deleu

Princeton University, Princeton NJ, USA

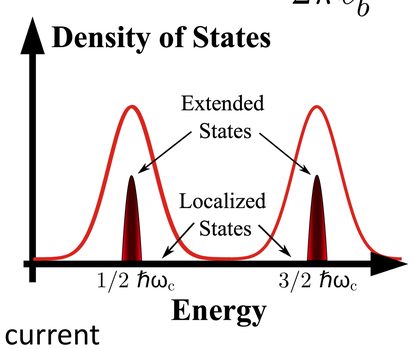
Advisor: Professor Ravindra N. Bhatt

Background

- Disorder shapes the topological properties of quantum systems, a crucial topic in modern physics
- A 2D system under high magnetic field splits into Landau levels with N_ϕ degeneracy
  - Disorder broadens these levels

$$N_\phi = \frac{L^2}{2\pi\ell_b^2}$$

- Extended states concentrate at band centers: these states conduct Hall current
  - **Quantum Hall Effect!**



- Chern number is a topological property of a state, quantifying its ability to carry Hall current
  - $C \neq 0$ states are extended, $C = 0$ states are localized
- Random matrix theory (RMT) describes the behavior of ensembles of random matrices
  - Hermitian matrices: Gaussian Unitary Ensemble (**GUE**)
- This work explores the distributions of eigenvalues and their spacings based on Chern number, connecting RMT & topology

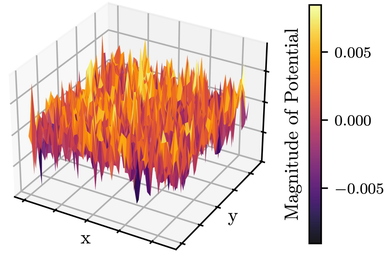
Approach

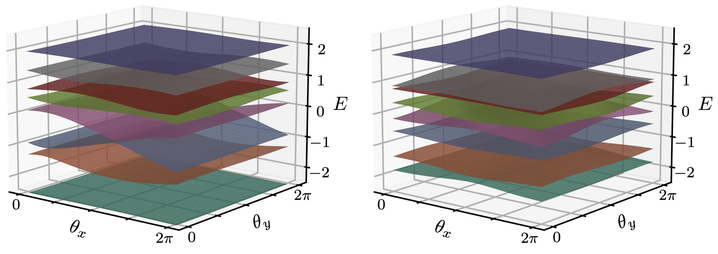
- Consider the high magnetic field limit, projecting to the lowest Landau level with a white noise potential
- Vary the area to change N_ϕ

$$H = \frac{1}{2m}(\mathbf{P} + \frac{e}{c}\mathbf{A})^2 + V(\mathbf{r})$$

Simulations

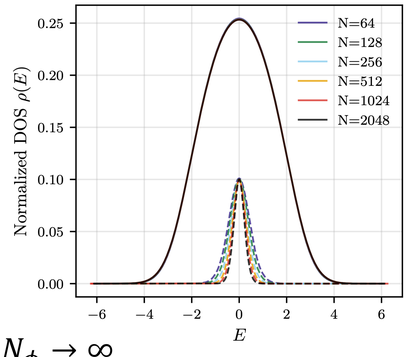
- Disorder from a a Gaussian white noise potential
- Simulations run in Python for 145k CPU-hours and 4k GPU hours for system sizes between 8 and 2048 states: 5-20 million eigenvalues collected per system





Energy spectra for 2 random $N_\phi = 8$ systems across boundary condition angles (above)

- All system-sizes exhibit similar global density of states
- As N_ϕ increases, the fraction of extended states decreases (dashed)
- Extended states collapse to $E = 0$ as $N_\phi \to \infty$

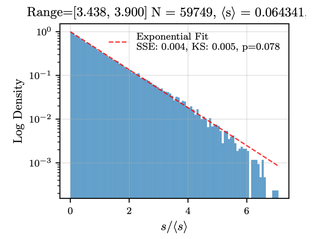


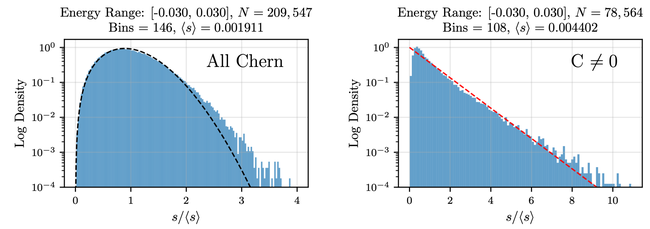
- Experimentally, the number of extended states as a function of system-size scales according to:

$$N_{C\neq 0} = 0.2607 N_\phi^{0.7986}$$

Results for N=2048

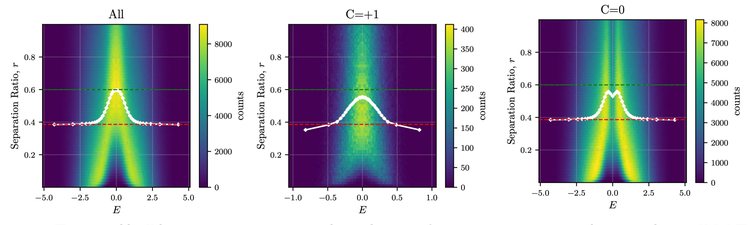
- Localized states at the edges of the energy spectrum have no level repulsions, following Poisson statistics for random variables (right)





- Separations of all states at band center are GUE-like, with a tail closer to e^{-s} rather than e^{-s^2} (left)
- Separations of extended states at the band center are Poisson-like with repulsions near s=0 (right)
  - Interspersed localized states dominate spacings here
- Another metric is the level-spacing ratio:

$$r_i = \frac{\min(E_i - E_{i-1}, E_{i+1} - E_i)}{\max(E_i - E_{i-1}, E_{i+1} - E_i)}$$



- For all Chern states, the band center reaches the GUE limit, and at the edges, the Poisson limit (dashed)
- The C=+1, C=0 cases approach the Poisson limit but not the GUE limit: there are intermediate results

While states collectively transition between GUE and Poisson statistics, distinct Chern states exhibit complex transitions, a promising direction for future exploration.